Bose symmetry interference effects of 4π final states

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We analyze the relative branching ratios of 4π final states $\pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0\pi^0$, and $\pi^0\pi^0\pi^0\pi^0$, from various resonances with $J^{PC}=0^{++}$, 0^{-+}, 2^{++}. We find that in some cases the interference effects due to Bose symmetry make their ratios obviously different from the naive counting values without these effects. The results allow an accurate estimate of the 4π branching ratios of relevant resonances when only one (or two) modes are measured in a given experiment.

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I. INTRODUCTION

In the mass range 1–2.5 GeV, there exist rich hadronic resonance spectra, including possible 0^{++}, 0^{-+}, and 2^{++} glueballs. Their decay branching ratios are an important source of information about the nature of these resonances. Among the observed decay modes for the 0^{++}, 0^{-+}, and 2^{++} resonances, the 4π final state is a very important one [1].

There are three kinds of 4π final states: $\pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0\pi^0$, and $\pi^0\pi^0\pi^0\pi^0$. Usually, because of the special features and limitations of each detector, one experiment is only good at studying one kind of 4π final state. For example, the Crystal Barrel detector is particularly good at studying neutral final states and has therefore studied resonances decaying into the $\pi^0\pi^0\pi^0\pi^0$ final state [2]; the BES (Beijing Spectrometer) detector is good at detecting charged particles and has hence studied J/ψ radiative decay to the $\pi^+\pi^-\pi^+\pi^-$ final state [3]. The problem is how to deduce from the measured rate for one kind of 4π final state the rates for the other two kinds of 4π final states, allowing for interference effects within the 4π channel.

The 4π final states are usually produced via 2-meson intermediate states M_1 and M_2:

$$M\to M_1+M_2\to 4\pi.$$

The parent mesons in which we are interested are $0^{++}, 0^{-+}$, and 2^{++}, etc., because they have the same quantum numbers as glueballs, which are under intensive discussions at present; M_1 and M_2 can be various mesons, among which $\sigma\sigma$, $\rho\rho$, and $f_2\sigma$ are the most likely candidates [2,3] and are discussed in this work.

Naive counting for the 4π final states from simple isospin decomposition would result in

$$\begin{aligned}&\Gamma(M\to\pi^+\pi^-\pi^0\pi^0):\Gamma(M\to\pi^+\pi^-\pi^+\pi^-):\\&\quad\Gamma(M\to\pi^0\pi^0\pi^0\pi^0)\\&\quad=4:4:1,\quad\text{for } f_2\sigma,\sigma\sigma \text{ intermediate states;}\end{aligned}\tag{1}$$

$$\begin{aligned}&\Gamma(M\to\pi^+\pi^-\pi^0\pi^0):\Gamma(M\to\pi^+\pi^-\pi^+\pi^-)\\&\quad:\Gamma(M\to\pi^0\pi^0\pi^0\pi^0)\\&\quad=2:1:0,\quad\text{for } \rho\rho \text{ intermediate states,}\end{aligned}\tag{2}$$

where all interference effects are neglected.

All pions can be treated as identical particles and each mode has the same production amplitude up to an SU(2) factor. So, unless all the interference terms cancel each other after integration over the invariant phase space of final states, their effects can be important. In the earlier literature, naive counting was employed to evaluate the relative production rate of one mode to another. Even though this counting way is simple and is valid in certain cases, it may cause large errors in some cases.

In this work, we analyze the relative ratios of $B(M\to\pi^+\pi^-\pi^0\pi^0):B(M\to\pi^+\pi^-\pi^+\pi^-):B(M\to\pi^0\pi^0\pi^0\pi^0)$ by including interference terms for various masses of the parent mesons of $0^{++}, 0^{-+}, 2^{++}$, and intermediate 2-meson states.

In Sec. II, we present the formulation of the analysis; numerical results and discussion are given in Sec. III.

II. FORMULATION

For $M\to\pi_1(p_1)\pi_2(p_2)\pi_3(p_3)\pi_4(p_4)$ where $p_i's$ $(i=1,\ldots,4)$ are the four-momenta of the four produced pions, we use the notation

$$p_{ab}\equiv p_a+p_b,\quad a,b=1,\ldots,4,\ \text{and}\ a\neq b.$$

The propagators take the Breit-Wigner form [4,5]

$$F_{ab}=\frac{-i}{p_{ab}^2-m^2+i\Gamma m}\quad \text{for scalar mesons,}$$

$$D_{ab}^{\alpha\beta}=\frac{i}{p_{ab}^2-m^2+i\Gamma m}\tilde{g}^{\alpha\beta}\quad \text{for massive vector,}$$

$$D_{ab}^{\alpha\beta\gamma\delta}=\frac{-i}{p_{ab}^2-m^2+i\Gamma m}\left[\frac{1}{2}(\tilde{g}^{\alpha\gamma}\tilde{g}^{\beta\delta}+\tilde{g}^{\alpha\delta}\tilde{g}^{\beta\gamma})-\frac{1}{3}\tilde{g}^{\alpha\beta}\tilde{g}^{\gamma\delta}\right]$$

$$\text{for spin-2 tensor meson,} \quad (3)$$

where

$$\tilde{g}^{\alpha\beta}\equiv -g^{\alpha\beta}+\frac{p_{ab}^{\alpha}p_{ab}^{\beta}}{m^2}, \quad (4)$$

and m is the mass of the intermediate meson of spin 0 or 1 or 2. In the following we present explicit expressions for the amplitude squared for the process $M\to M_1+M_2\to 4\pi$ with various M,M_1,M_2 identities. For each single channel, since the common vertex coupling constants can be absorbed into an overall normalization constant, we do not write them out explicitly in the formulas. In this work, we consider only the relative values of the branching ratios and the overall normalization constant is irrelevant here.

A. Decay of $M(0^{++})\to 4\pi$

To investigate interference effects for $M\to 4\pi$, we distinguish the processes caused by different intermediate states. Here we first ignore possible interference between $M\to\sigma\sigma\to 4\pi$ and $M\to\rho\rho\to 4\pi$ and will study it later.

(a) The squares of amplitudes corresponding to $\sigma\sigma$ intermediate state are

$$|M|^2=\frac{1}{2}|F_{12}^{\sigma}F_{34}^{\sigma}|^2 \quad \text{for } f_0\to\pi^+\pi^-\pi^0\pi^0,$$

$$|M|^2=\frac{1}{4}|F_{12}^{\sigma}F_{34}^{\sigma}+F_{14}^{\sigma}F_{32}^{\sigma}|^2 \quad \text{for } f_0\to\pi^+\pi^-\pi^+\pi^-,$$

$$|M|^2=\frac{1}{24}|F_{12}^{\sigma}F_{34}^{\sigma}+F_{13}^{\sigma}F_{24}^{\sigma}+F_{14}^{\sigma}F_{32}^{\sigma}|^2$$

$$\text{for } f_0\to\pi^0\pi^0\pi^0\pi^0. \quad (5)$$

(b) For $\rho\rho$ intermediate states:

$$|M|^2=\frac{1}{2}|(p_1-p_2)\cdot(p_3-p_4)F_{12}^{\rho}F_{34}^{\rho}+(p_1-p_4)\cdot(p_3-p_2)F_{14}^{\rho}F_{32}^{\rho}|^2$$

$$\text{for } f_0\to\pi^+\pi^0\pi^-\pi^0,$$

$$|M|^2=\frac{1}{4}|(p_1-p_2)\cdot(p_3-p_4)F_{12}^{\rho}F_{34}^{\rho}+(p_1-p_4)\cdot(p_3-p_2)F_{14}^{\rho}F_{32}^{\rho}|^2$$

$$\text{for } f_0\to\pi^+\pi^-\pi^+\pi^-, \quad (6)$$

where $F_{ij}^{\sigma},F_{ij}^{\rho}$ are the propagators of the σ meson and ρ meson, respectively, for pions i and j. There is no process $f_0\to\rho\rho\to 4\pi^0$, because $\rho^0\to\pi^0\pi^0$ is forbidden by isospin symmetry.

B. Decay of 0^{-+} mesons

It is obvious that $0^{-+}\to\sigma\sigma$ is forbidden by parity and angular-momentum conservations; it decays into 4π only via $\rho\rho$ intermediate states.

$$|M|^2=\frac{1}{2}|\epsilon_{\mu\nu\lambda\rho}(p_1^{\mu}p_2^{\nu}p_3^{\lambda}p_4^{\rho}F_{12}^{\rho}F_{34}^{\rho}-p_1^{\mu}p_4^{\nu}p_3^{\lambda}p_2^{\rho}F_{14}^{\rho}F_{32}^{\rho})|^2$$

$$\text{for } 0^{-+}\to\pi^+\pi^0\pi^-\pi^0,$$

$$|M|^2=\frac{1}{4}|\epsilon_{\mu\nu\lambda\rho}(p_1^{\mu}p_2^{\nu}p_3^{\lambda}p_4^{\rho}F_{12}^{\rho}F_{34}^{\rho}-p_1^{\mu}p_4^{\nu}p_3^{\lambda}p_2^{\rho}F_{14}^{\rho}F_{32}^{\rho})|^2$$

$$\text{for } 0^{-+}\to\pi^+\pi^-\pi^+\pi^-. \quad (7)$$

C. Decay of 2^{++} mesons

For $2^{++}\to 4\pi$, in addition to the $\sigma\sigma$ and $\rho\rho$ intermediate states, the $f_2(1270)\sigma$ intermediate state is also found to be important.

(a) For $\sigma\sigma$ intermediate states:

$$|M|^2=\frac{1}{2}\left|\sqrt{\frac{1}{6}}(-\mathbf{r}^2+3r_Z^2)F_{12}^{\sigma}F_{34}^{\sigma}\right|^2$$

$$\text{for } 2^{++}\to\pi^+\pi^-\pi^0\pi^0,$$

$$|M|^2=\frac{1}{4}\left|\sqrt{\frac{1}{6}}(-\mathbf{r}^2+3r_Z^2)F_{12}^{\sigma}F_{34}^{\sigma}+\sqrt{\frac{1}{6}}(-\mathbf{r}''^2+3r_Z''^2)F_{14}^{\sigma}F_{32}^{\sigma}\right|^2$$

$$\text{for } 2^{++}\to\pi^+\pi^-\pi^+\pi^-,$$

$$|M|^2=\frac{1}{24}g''^2\left|\sqrt{\frac{1}{6}}(-\mathbf{r}^2+3r_Z^2)F_{12}^{\sigma}F_{34}^{\sigma}+\sqrt{\frac{1}{6}}(-\mathbf{r}'^2+3r_Z'^2)F_{13}^{\sigma}F_{24}^{\sigma}+\sqrt{\frac{1}{6}}(-\mathbf{r}''^2+3r_Z''^2)F_{14}^{\sigma}F_{32}^{\sigma}\right|^2$$

$$\text{for } 2^{++}\to\pi^0\pi^0\pi^0\pi^0, \quad (8)$$

where $r \equiv p_{12} - p_{34}$, $r' \equiv p_{13} - p_{24}$, $r'' \equiv p_{14} - p_{32}$, and r_Z, r'_Z, r''_Z are the z components of the three vectors $\mathbf{r}$, $\mathbf{r}'$, $\mathbf{r}''$, respectively.

(b) For $\rho\rho$ intermediate states:

$$|M|^2 = \frac{1}{2} |(-p^1_{12}p^1_{34} - p^2_{12}p^2_{34} + 2p^3_{12}p^3_{34})F^\rho_{12}F^\rho_{34} + (-p^1_{14}p^1_{32} - p^2_{14}p^2_{32} + 2p^3_{14}p^3_{32})F^\rho_{14}F^\rho_{32}|^2 \quad \text{for } 2^{++} \to \pi^+\pi^0\pi^-\pi^0,$$

$$|M|^2 = \frac{1}{4} |(-p^1_{12}p^1_{34} - p^2_{12}p^2_{34} + 2p^3_{12}p^3_{34})F^\rho_{12}F^\rho_{34} + (-p^1_{14}p^1_{32} - p^2_{14}p^2_{32} + 2p^3_{14}p^3_{32})F^\rho_{14}F^\rho_{32}|^2 \quad \text{for } 2^{++} \to \pi^+\pi^-\pi^+\pi^-, \quad (9)$$

where p^1_{ab}, p^2_{ab}, p^3_{ab} are the x,y,z components of the p_{ab}, respectively.

(c) For $f_2\sigma$ intermediate states:

$$|M|^2 = \frac{1}{2} |(-T^{11}_{(12)} - T^{22}_{(12)} + 2T^{33}_{(12)})F^{f_2}_{12}F^\sigma_{34}|^2 \quad \text{for } 2^{++} \to f_2\sigma \to \pi^+\pi^-\pi^0\pi^0,$$

$$|M|^2 = \frac{1}{4} |(-T^{11}_{(12)} - T^{22}_{(12)} + 2T^{33}_{(12)})F^{f_2}_{12}F^\sigma_{34} + (-T^{11}_{(14)} - T^{22}_{(14)} + 2T^{33}_{(14)})F^{f_2}_{14}F^\sigma_{32}|^2 \quad \text{for } 2^{++} \to f_2\sigma \to \pi^+\pi^-\pi^+\pi^-,$$

$$|M|^2 = \frac{1}{24} |(-T^{11}_{(12)} - T^{22}_{(12)} + 2T^{33}_{(12)})F^{f_2}_{12}F^\sigma_{34} + (-T^{11}_{(13)} - T^{22}_{(13)} + 2T^{33}_{(13)})F^{f_2}_{13}F^\sigma_{24} + (-T^{11}_{(14)} - T^{22}_{(14)} + 2T^{33}_{(14)})F^{f_2}_{14}F^\sigma_{32}|^2 \quad \text{for } 2^{++} \to f_2\sigma \to \pi^0\pi^0\pi^0\pi^0, \quad (10)$$

where

$$T^{ii}_{(ab)} = q^i_{ab}q^i_{ab} + \frac{1}{3}(1 + p^i_{ab}p^i_{ab}/M^2_{f_2})|\vec{q}_{ab}|^2, \quad (11)$$

and

$$q_{ab} = p_a - p_b, \quad F^{f_2} = \frac{1}{M^2_{f_2} - s_{ab} - iM_{f_2}\Gamma_{f_2}}, \quad s_{ab} = p^2_{ab}.$$

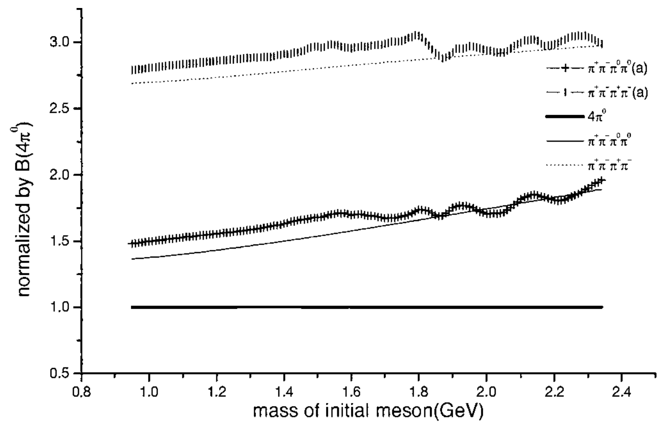

FIG. 1. Branching ratios for $0^{++} \to \sigma\sigma \to 4\pi$, relative to $4\pi^0$.

D. The interference between channels with $\sigma\sigma$ and $\rho\rho$ intermediate states in $M \to 4\pi$

So far we have ignored possible interference between $\sigma\sigma$ and $\rho\rho$ channels. In this subsection, we study this interference. As an example, we concentrate on 0^{++} decays. The squares of amplitudes are

$$|M|^2 = \frac{1}{2} |g'(p_1 - p_2)\cdot(p_3 - p_4)F^\rho_{12}F^\rho_{34} + g'(p_1 - p_4)\cdot(p_3 - p_2)F^\rho_{14}F^\rho_{32} + gF^\sigma_{13}F^\sigma_{24}|^2 \quad \text{for } f_0 \to \pi^+\pi^0\pi^-\pi^0,$$

$$|M|^2 = \frac{1}{4} |g'(p_1 - p_2)\cdot(p_3 - p_4)F^\rho_{12}F^\rho_{34} + g'(p_1 - p_4)\cdot(p_3 - p_2)F^\rho_{14}F^\rho_{32} + g(F^\sigma_{12}F^\sigma_{34} + F^\sigma_{14}F^\sigma_{32})|^2 \quad \text{for } f_0 \to \pi^+\pi^-\pi^+\pi^-, \quad (12)$$

where

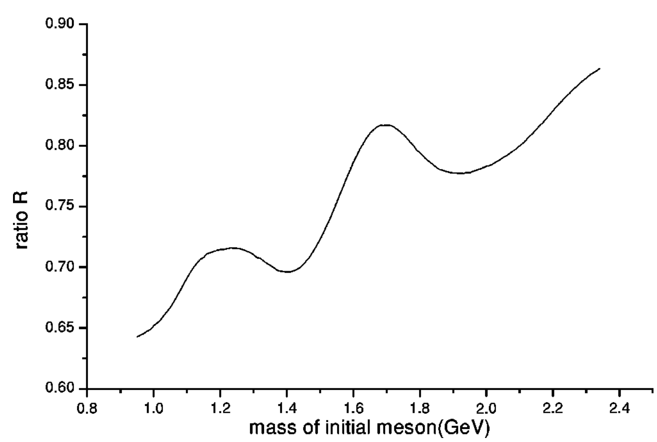

FIG. 2. Ratio of cross sections with and without the Bose symmetry interference effect for $0^{++} \to \rho\rho \to 4\pi$.

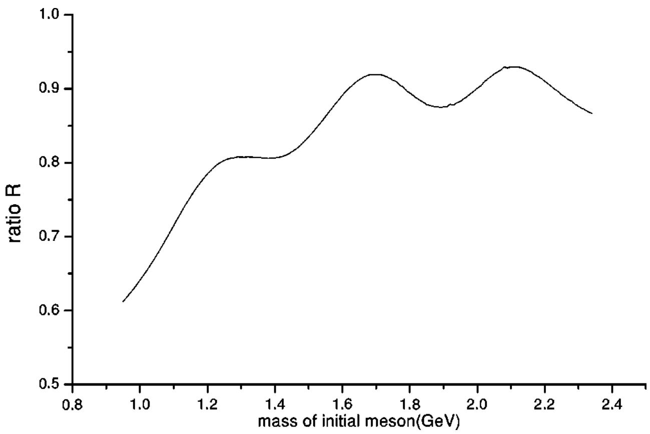

FIG. 3. Ratio of cross sections with and without the Bose symmetry interference effect for $0^{-+}\to\rho\rho\to 4\pi$.

$$g\equiv g_f\cdot g_\sigma^2,\quad g'\equiv g_f'\cdot g_\rho^2 \tag{13}$$

with g_f, g_f', g_ρ, and g_σ related to the partial decay widths as follows:

$$\Gamma(f_0\to\sigma\sigma)=\frac{g_f^2}{16\pi M_f}\left(1-\frac{4m_\sigma^2}{M_f^2}\right)^{1/2},$$

$$\Gamma(f_0\to\rho\rho)=\frac{g_f'^2}{16\pi M_f}\left(1-\frac{4m_\rho^2}{M_f^2}\right)^{1/2}\times\left[3+\frac{1}{4m_\rho^4}(M_f^4-4M_f^2m_\rho^2)\right],$$

$$\Gamma(\rho\to 2\pi)=\frac{g_\rho^2 m_\rho}{48\pi}\left(1-\frac{4m_\pi^2}{m_\rho^2}\right)^{3/2},$$

$$\Gamma(\sigma\to\pi\pi)=\frac{g_\sigma^2}{16\pi m_\sigma}\left(1-\frac{4m_\pi^2}{m_\sigma^2}\right)^{1/2}. \tag{14}$$

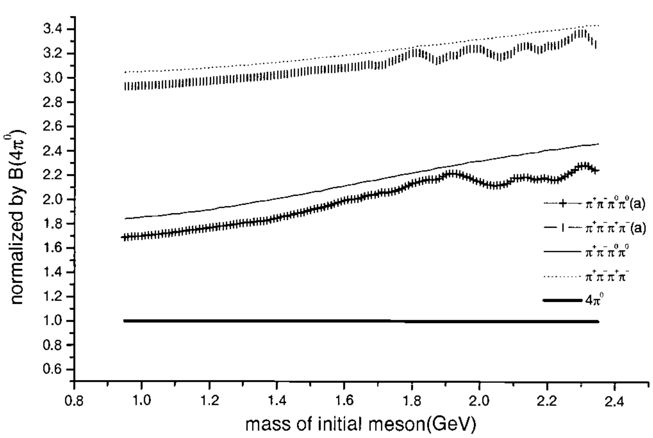

FIG. 4. Branching ratios for $2^{++}\to\sigma\sigma\to 4\pi$, relative to $4\pi^0$.

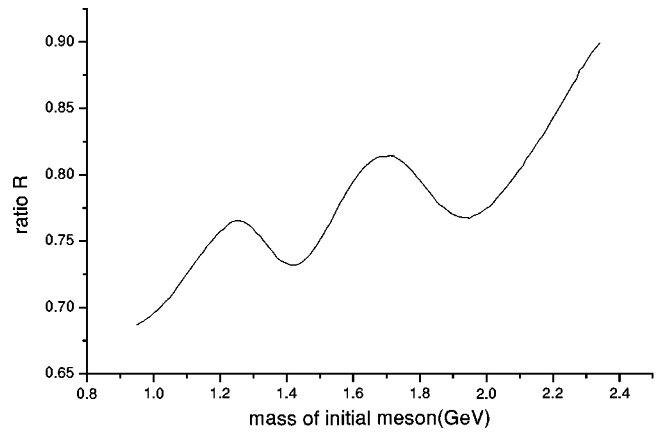

FIG. 5. Ratio of cross sections with and without the Bose symmetry interference effect for $2^{++}\to\rho\rho\to 4\pi$.

III. NUMERICAL RESULTS AND DISCUSSION

We have employed a Monte Carlo program to carry out the calculation of the various 4π decay widths. There, we need to multiply the propagators of vector and tensor F^ρ and F^{f_2} in all equations by the Blatt-Weisskopt barrier factor $B_l(p)$ [5,7] ($B_0(p)=1$), which is widely used in partial-wave analyses. Explicitly F^ρ is replaced by $F^\rho B_1(p)$ and F^{f_2} by $F^{f_2}B_2(p)$, respectively. We also include the corresponding centrifugal barrier factors for the vertices of the parent mesons to the two-intermediate mesons.

For the σ propagator, there are various forms. Here we only use two typical ones. The first is [8]

$$F^\sigma=\frac{1}{M_\sigma^2-s-iM_\sigma[\Gamma_1(s)+\Gamma_2(s)]}, \tag{15}$$

where

$$\Gamma_1(s)=G_1\frac{\sqrt{1-4m_\pi^2/s}}{\sqrt{1-4m_\pi^2/M_\sigma^2}}\,\frac{s-m_\pi^2/2}{M_\sigma^2-m_\pi^2/2}e^{-(s-M_\sigma^2)/4\beta^2},$$

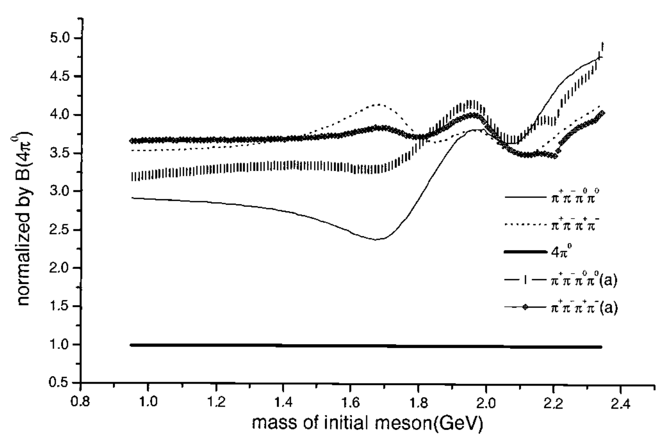

FIG. 6. Branching ratios for $2^{++}f_2\sigma\to 4\pi$, relative to $4\pi^0$.

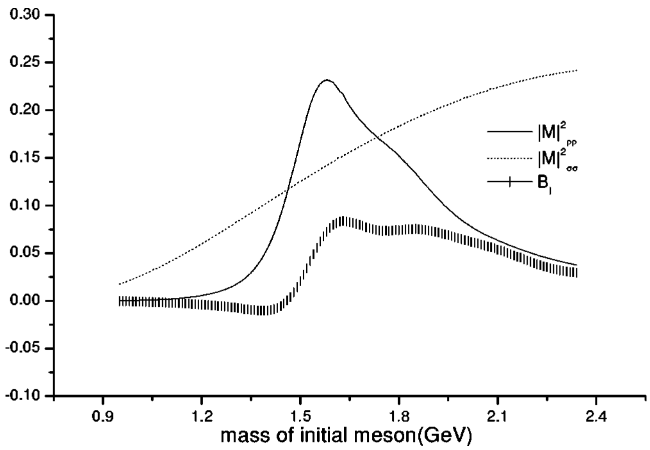

FIG. 7. The total squares of amplitudes for $\pi^+\pi^-\pi^0\pi^0$ final states including interference between $\sigma\sigma$ and $\rho\rho$.

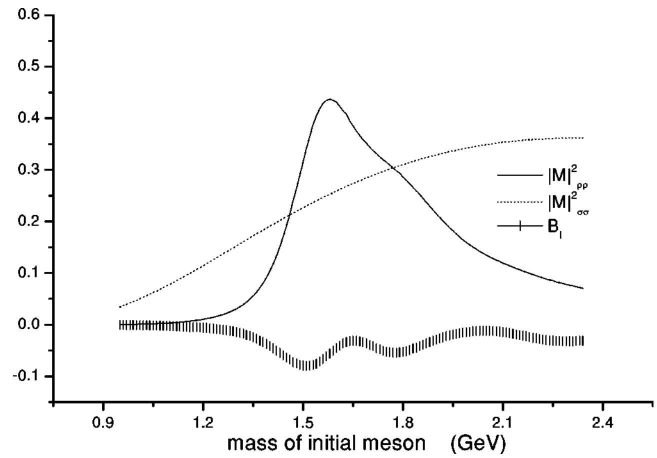

FIG. 8. The total squares of amplitudes for $\pi^+\pi^-\pi^+\pi^-$ final states including interference between $\sigma\sigma$ and $\rho\rho$.

$$\Gamma_2(s)=G_2\frac{\sqrt{1-16m_\pi^2/s}}{1+\exp[\Lambda(s_0-s)]}\cdot\frac{1+\exp[\Lambda(s_0-M_\sigma^2)]}{\sqrt{1-16m_\pi^2/M_\sigma^2}} \tag{16}$$

with $M_\sigma=1.067$ GeV, $G_1=1.378$ GeV, $\beta=0.7$ GeV, $G_2=0.0036$ GeV, $\Lambda=3.5$ GeV^{-2}, and $s_0=2.8$ GeV^2.

The second is [9],

$$F^\sigma=\frac{e^{2i\phi}-1}{2i}+\frac{g_1\rho_1 e^{2i\phi}}{M_R^2-s-i(\rho_1 g_1+\rho_2 g_2)}, \tag{17}$$

with

$$e^{2i\phi}=\frac{1+a_1 s+a_2 s^2+i\rho_1[b_1(s-M_\pi^2/2)+b_2 s^2]}{1+a_1 s+a_2 s^2-i\rho_1[b_1(s-M_\pi^2/2)+b_2 s^2]}, \tag{18}$$

where $a_1=-0.3853$ GeV^{-2}, $a_2=-0.4237$ GeV^{-4}, $b_1=-3.696$ GeV^{-2}, $b_2=-1.462$ GeV^{-4}, $g_1=0.1108$, $g_2=0.4229$, $M_R=0.9535$ GeV, $\rho_1=\sqrt{1-4m_\pi^2/s}$, $\rho_2=\sqrt{1-4m_\kappa^2/s}$, and s is the invariant mass squared of the system. This parametrization is the full $\pi\pi$ S-wave scattering amplitude corresponding to CERN-Münich $\pi\pi$ S-wave phase shifts [10] and is very close to the AMP amplitude [11], and hence includes contributions from several 0^{++} resonances.

We now present our results graphically and add some discussion.

(1) In Fig. 1 we present the relative branching ratios of $B(0^{++}\to\sigma\sigma\to 4\pi)$ normalized to $B(0^{++}\to\sigma\sigma\to 4\pi^0)$. Clearly the ratios $B(\pi^+\pi^-\pi^0\pi^0):B(\pi^+\pi^-\pi^+\pi^-)$: $B(\pi^0\pi^0\pi^0\pi^0)$ deviate from the naive counting 4:4:1. The curves $\pi^+\pi^-\pi^0\pi^0(a)$ and $\pi^+\pi^-\pi^+\pi^-(a)$ are evaluated using Eq. (15) for the σ propagator, while the others correspond to Eq. (17); the latter has a more complicated structure than Eq. (15) and causes the curly structures. The same conventions apply to Fig. 4 and Fig. 6.

(2) To study the significance of the interference effects in $M\to\rho\rho\to 4\pi$, we define a quantity R as

$$R=\frac{\int[\text{LIPS}]\sum_i |A_i|^2}{\int[\text{LIPS}]\left|\sum_i A_i\right|^2}, \tag{19}$$

where the sum runs over all channels which contribute to the same 4π final state; the channels may interfere with each other and the integration is carried out over the Lorentz invariant phase-space (LIPS). If the interference effects are negligible, R should be equal to unity. If the interference is destructive $R>1$, whereas for the constructive case $R<1$.

Figure 2 gives the dependence of the R value on the mass of the initial 0^{++} meson for $0^{++}\to\rho\rho\to\pi^+\pi^-\pi^0\pi^0$ and $0^{++}\to\rho\rho\to\pi^+\pi^-\pi^+\pi^-$. The figure demonstrates constructive interference. The R value increases with the mass and approaches unity for sufficiently high mass. This is consistent with our understanding that at very high energies, interference would become less and less significant.

(3) Figure 3 is for $0^{-+}\to\rho\rho\to 4\pi$ whose structure is analogous to Fig. 2.

TABLE I. Branching ratios of J/ψ radiative decays to 4π based on the BES data.

Parent meson	$B(\pi^+\pi^-\pi^+\pi^-)$ (measured)	$B(\pi^+\pi^-\pi^0\pi^0)$	$B(\pi^0\pi^0\pi^0\pi^0)$	$B(4\pi)$
$f_0(1500)$	$(3.1\pm0.2\pm1.1)\times10^{-4}$	1.75×10^{-4}	1.05×10^{-4}	5.9×10^{-4}
$f_0(1740)$	$(3.1\pm0.2\pm1.1)\times10^{-4}$	1.73×10^{-4}	1.03×10^{-4}	5.9×10^{-4}
$f_0(2100)$	$(5.1\pm0.3\pm1.8)\times10^{-4}$	3.08×10^{-4}	1.71×10^{-4}	9.9×10^{-4}
$f_2(1950)$	$(5.5\pm0.3\pm1.9)\times10^{-4}$	5.58×10^{-4}	1.63×10^{-4}	12.7×10^{-4}

TABLE II. Branching ratios of J/ψ radiative decays to 4π based on the MARK III data.

Parent meson	$B(\pi^+\pi^-\pi^+\pi^-)$ (measured)	$B(\pi^+\pi^-\pi^0\pi^0)$	$B(\pi^0\pi^0\pi^0\pi^0)$	$B(4\pi)$
$f_0(1505)$	$(2.5\pm0.4)\times10^{-4}$	1.41×10^{-4}	0.84×10^{-4}	4.8×10^{-4}
$f_0(1750)$	$(4.3\pm0.6)\times10^{-4}$	2.41×10^{-4}	1.43×10^{-4}	8.1×10^{-4}
$f_0(2104)$	$(3.0\pm0.8)\times10^{-4}$	1.82×10^{-4}	1.00×10^{-4}	5.8×10^{-4}

(4) Figure 4 is for $2^{++}\to\sigma\sigma\to4\pi$ whose structure is in analog to Fig. 1. Figure 5 is for $2^{++}\to\rho\rho\to4\pi$ whose structure is generally similar to that of Figs. 2 and 3. In Figs. 2, 3, and 5 for $\rho\rho$ intermediate states, there are several peaks, while in Figs. 1 and 4 for $\sigma\sigma$ intermediate states the curves go up steadily as the mass increases. This is due to the more complicated momentum-dependent vertices for the $\rho\rho$ case.

(5) Figure 6 shows the relative branching ratios of $2^{++}\to f_2\sigma\to\pi^+\pi^-\pi^0\pi^0$, $\pi^+\pi^-\pi^+\pi^-$, $\pi^0\pi^0\pi^0\pi^0$, respectively, normalized to $B(2^{++}\to\pi^0\pi^0\pi^0\pi^0)$. The interference effects here are smaller than in the case of $\sigma\sigma$ intermediate state, but still not negligible.

(6) As mentioned above, we deliberately ignore interference among different intermediate channels. This will be true if one of the channels dominates over the other. Here we study the interference between $\sigma\sigma$ and $\rho\rho$ intermediate channels in $0^{++}\to4\pi$ decays. This estimation depends on the effective couplings g' and g. In Figs. 7 and 8 we deal with the $\pi^+\pi^-\pi^0\pi^0$ and $\pi^+\pi^-\pi^+\pi^-$ final states, respectively. For convenience, we compute $|M|^2_{\sigma\sigma}$ according to Eq. (5) and $|M|^2_{\rho\rho}$ according to Eq. (6), then obtain the interference term as $B_I=(|M|^2-|M|^2_{\rho\rho}-|M|^2_{\sigma\sigma})$ for various masses of the parent meson; the formula for $|M|^2$ is given in Eq. (12). In Fig. 7, g'/g is taken to be 4.5, and 8.5 in Fig. 8. We choose the form (15) for the σ propagator.

For the $f_0(1500)$, $f_0(1750)$ and $f_0(2100)$, the $\sigma\sigma$ intermediate state dominates over the $\rho\rho$ intermediate states [3,6]. Hence the interference between $\sigma\sigma$ and $\rho\rho$ intermediate states is negligible for these states.

Now we apply our results to estimate the branching ratios of channels which are not yet measured. We tabulate numerical results for some branching ratios of J/ψ radiative decays to 4π states where only one of the three 4π modes $(\pi^+\pi^-\pi^+\pi^-)$ is experimentally measured. In Table I, the first column contains values of $B(\pi^+\pi^-\pi^+\pi^-)$ measured by the BES Collaboration [3], while the other two columns are for the ones evaluated in terms of our scheme where interference effects are included. Table II is similar but based on the Mark III data [6].

Using Crystal Barrel results [2] and our results here, we find the relative branching ratio of $\mathrm{Br}(f_0(1500)\to4\pi)/\mathrm{Br}(f_0(1500)\to2\pi)$ to be (2.1 ± 0.6); this is compatible with the result 1.5 ± 0.4 from $\pi\pi$ scattering phase shifts [8]. It corrects the value 3.4 ± 0.8 [2] obtained by ignoring interferences.

In conclusion, our numerical results indicate that interference effects make the ratios

$$B(M\to\pi^+\pi^-\pi^0\pi^0):B(M\to\pi^+\pi^-\pi^+\pi^-):B(M\to\pi^0\pi^0\pi^0\pi^0)$$

deviate significantly from the naive counting 4:4:1 for isoscalar 0^{++} and 2^{++} mesons. The graphs provided in this work can serve as a standard reference; once one of the 4π modes $\pi^+\pi^-\pi^0\pi^0,\pi^+\pi^-\pi^+\pi^-,\pi^0\pi^0\pi^0\pi^0$ is measured, we can determine the branching ratios of the other modes.

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